

TRIGONOMETRIC IDENTITIES

Reciprocal and Quotient Identities:

$$\begin{aligned}\csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} \\ \sin \theta &= \frac{1}{\csc \theta} & \cos \theta &= \frac{1}{\sec \theta} & \tan \theta &= \frac{1}{\cot \theta} \\ & & \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta}\end{aligned}$$

Cofunction Identities:

$$\begin{aligned}\sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta & \cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta \\ \tan\left(\frac{\pi}{2} - \theta\right) &= \cot \theta & \cot\left(\frac{\pi}{2} - \theta\right) &= \tan \theta \\ \sec\left(\frac{\pi}{2} - \theta\right) &= \csc \theta & \csc\left(\frac{\pi}{2} - \theta\right) &= \sec \theta\end{aligned}$$

Pythagorean Identities:

$$\begin{aligned}\cos^2 \theta + \sin^2 \theta &= 1 \\ 1 + \tan^2 \theta &= \sec^2 \theta \\ \cot^2 \theta + 1 &= \csc^2 \theta\end{aligned}$$

Odd-Even Identities:

$$\begin{aligned}\sin(-x) &= -\sin x & \cos(-x) &= \cos x & \tan(-x) &= -\tan x \\ \csc(-x) &= -\csc x & \sec(-x) &= \sec x & \cot(-x) &= -\cot x\end{aligned}$$

Sum or Difference Identities:

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

Double Angle Identities:

$$\begin{aligned}\sin 2u &= 2 \sin u \cos u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \\ \cos 2u &= \cos^2 u - \sin^2 u \\ \cos 2u &= 2 \cos^2 u - 1 \\ \cos 2u &= 1 - 2 \sin^2 u\end{aligned}$$

Half-Angle Formulas:

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

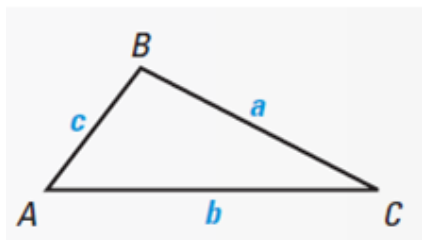
$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$$

Product to sum formulas:

$$\begin{aligned}\sin x \sin y &= \frac{1}{2} [\cos(x - y) - \cos(x + y)] \\ \cos x \cos y &= \frac{1}{2} [\cos(x - y) + \cos(x + y)] \\ \sin x \cos y &= \frac{1}{2} [\sin(x + y) + \sin(x - y)] \\ \cos x \sin y &= \frac{1}{2} [\sin(x + y) - \sin(x - y)]\end{aligned}$$

Sum to product formulas:

$$\begin{aligned}\sin x + \sin y &= 2 \sin \left(\frac{x+y}{2}\right) \cos \left(\frac{x-y}{2}\right) \\ \sin x - \sin y &= 2 \cos \left(\frac{x+y}{2}\right) \sin \left(\frac{x-y}{2}\right) \\ \cos x + \cos y &= 2 \cos \left(\frac{x+y}{2}\right) \cos \left(\frac{x-y}{2}\right) \\ \cos x - \cos y &= -2 \sin \left(\frac{x+y}{2}\right) \sin \left(\frac{x-y}{2}\right)\end{aligned}$$



Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right) = A$$

$$b^2 = a^2 + c^2 - 2ac \cos B \quad \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right) = B$$

$$c^2 = b^2 + a^2 - 2ba \cos C \quad \cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right) = C$$

Area of Oblique Triangle

Heron's Area Formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$, $s = \frac{a+b+c}{2}$

Area = $\frac{1}{2} bc \sin A$, $\frac{1}{2} ac \sin B$, $\frac{1}{2} ab \sin C$

Angle between two vectors: $\cos \theta = \frac{u \cdot v}{\|u\| \|v\|}$ or $\sin \theta = \frac{\|u \times v\|}{\|u\| \|v\|}$

Unit Vector: $\frac{u}{\|u\|}$

Complex numbers:

$$z = r(\cos \theta + i \sin \theta) \quad a = r \cos \theta \quad b = r \sin \theta \quad r = \sqrt{a^2 + b^2} \quad \tan \theta = \frac{b}{a}$$

$$z_1 \cdot z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} (\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2))$$

$$\text{De Moivre's Thrm: } z^n = [r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

$$\text{Roots of a complex number: } \sqrt[n]{r} \left(\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n} \right)$$

$$\text{Equation of a line: } x = x_1 + at$$

$$\text{Equation of a Plane: } a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$y = y_1 + bt$$

$$z = z_1 + ct$$